SOME MAGNETOHYDRODYNAMIC PROBLEMS IN LONGITUDINAL FLOW OVER A POROUS CYLINDRICAL SURFACE

(NEKOTORYE MAGNITOGIDRODINAMICHESKIE ZADACHI O PRODOL'NOM OBTEKANII PRONITSAEMOI TSILINDRICHESKOI POVERKHNOSTI)

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One of the methods for controlling a boundary layer is sucking or blowing of the fluid through the surface. If the boundary layer is electrically conducting, then there is also the possibility of using magnetic and electric fields for the same purpose. In connection with this, it is interesting to investigate the effects which appear in a combination of both methods.

An approximate solution of this type was found by Vatazhin [1], who investigated the flow of a viscous compressible gas around a semi-infinite plate, with blowing and a transverse magnetic field. A very similarly posed problem was also studied by Lykoudis [2,3], but his papers differ from [1] in that the gas is taken to be conducting throughout, and not only in the boundary layer.

In the solutions mentioned, the influence of the induced component of magnetic field along the surface was neglected. Therefore, within the framework of such a theory, it is not possible to include magnetohydrodynamic effects which arise from the interaction of the transverse (normal to the surface) flow with the induced field. For studying these effects it is necessary to proceed either from the full system of equations of magnetohydrodynamics or from the boundary-layer equations developed by Zhigulev [4]. The solutions of some particular solutions of similar type are published in the papers of Gupta [5], Yasuhara [6], and Greenspan [7].

In the present paper an attempt has been made to ascertain the possibility of a more general approach to problems of longitudinal flow past porous cylindrical surfaces, similar to what has been done for internal and external rectilinear flows [8,9].

1. Let us consider a porous cylindrical surface S in a basic flow in the direction of a generator. We will assume that the contour Σ of the cross-section of the surface is smooth and closed, and that the flow is external.

If the flow is stationary, the fluid incompressible, and its physical properties constant, then the velocity, the magnetic field and the pressure in the flow are determined from the equations

$$\rho\,(\mathbf{V}\nabla)\,\mathbf{V} = -\nabla p^* + \varkappa\,(\mathbf{H}\nabla)\,\mathbf{H} + \eta\Delta\mathbf{V}, \qquad \operatorname{div}\mathbf{V} = 0$$
$$(\mathbf{V}\nabla)\,\mathbf{H} = (\mathbf{H}\nabla)\,\mathbf{V} + \nu_m\Delta\mathbf{H}, \qquad \operatorname{div}\mathbf{H} = 0 \qquad (1.1)$$

where $p^* = p + \kappa H^2/2$, $\kappa = \mu/4\pi$, $\nu_m = c^2/4\pi\mu\sigma$, and the remaining symbols are well known. The electric field and the current density are found from the relations

$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\,\mathbf{j}, \qquad \mathbf{j} = \sigma\left(\mathbf{E} + \frac{\mu}{c}\,\mathbf{V}\times\mathbf{H}\right) \qquad (1.2)$$

As an initial assumption, we shall take the velocity components and the components of the magnetic field normal to the basic flow to be constant along a generator of the surface. Then, putting the z-axis parallel to a generator, we may write

$$\mathbf{V} = \mathbf{V}_\perp + v\mathbf{e}_z, \qquad \mathbf{V}_\perp = v_x\,(x, y)\,\mathbf{e}_x + v_y\,(x, y)\,\mathbf{e}_y, \qquad v = v_z(x, y)$$
$$\mathbf{H} = \mathbf{H}_\perp + h\mathbf{e}_z, \qquad \mathbf{H}_\perp = H_x\,(x, y)\,\mathbf{e}_x + H_y\,(x, y)\,\mathbf{e}_y, \qquad h = H_z\,(x, y, z)$$

Putting this into Equations (1.1) and projecting the first and third of them on the xy-plane and the z-axis, we obtain

$$\rho\,(\mathbf{V}_\perp\nabla)\,\mathbf{V}_\perp = -\nabla' p^* + \varkappa\,(\mathbf{H}_\perp\nabla)\,\mathbf{H}_\perp + \eta\Delta\mathbf{V}_\perp \qquad \left(\nabla' = \mathbf{e}_x\frac{\partial}{\partial x} + \mathbf{e}_y\frac{\partial}{\partial y}\right) \qquad (1.3)$$

$$\rho\mathbf{V}_\perp\nabla v = -\frac{\partial p^*}{\partial z} + \varkappa\mathbf{H}_\perp\nabla h + \varkappa h\frac{\partial h}{\partial z} + \eta\Delta v \qquad (1.4)$$

$$(\mathbf{V}_\perp\nabla)\,\mathbf{H}_\perp = (\mathbf{H}_\perp\nabla)\,\mathbf{V}_\perp + \nu_m\Delta\mathbf{H}_\perp \qquad (1.5)$$

$$\mathbf{V}_\perp\nabla h + v\frac{\partial h}{\partial z} = \mathbf{H}_\perp\nabla v + \nu_m\Delta h \qquad (1.6)$$

$$\operatorname{div}\mathbf{V}_\perp = 0 \qquad (1.7)$$

$$\operatorname{div}\mathbf{H}_\perp + \frac{\partial h}{\partial z} = 0 \quad \text{or} \quad \operatorname{div}\mathbf{H}_\perp = -\theta\,(x, y), \quad h = z\theta\,(x, y) + h_0(x, y) \qquad (1.8)$$

from which it immediately follows from (1.3), (1.4) and (1.8) that $\partial^2 p^*\,\partial z^2 = \text{const}$.

It is necessary also to formulate the equations describing the flow and the electromagnetic field inside the body. If it is assumed that the body can move only in the direction of its axis with constant velocity v_w, and that the vectors of the transverse velocity and transverse magnetic field for the region inside the body do not depend on z, then the following cases can occur.

a) Flow over a porous body with internal sources of fluid. The magnetic field is described by equations of the type (1.5), (1.6) and (1.8) where $v = v_w$, and $\mathbf{V}_\perp$ is either given or is determined with the help of auxiliary equations, for example, magnetohydrodynamic equations of the type (1.3), (1.7) or equations of the theory of filtration.

b) Flow over a non-porous body having a continuous distribution of sources on its surface. The magnetic field inside the body is described by Maxwell's equations.

On the surface of the body, the usual conditions on the velocity and the magnetic and electric fields have to be satisfied:

$$v = v_w, \quad v_\tau = 0, \quad [v_n] = q, \quad [\mu H_n] = 0, \quad [E_z] = 0$$
$$[E_\tau] = 0, \quad [h] = 0, \quad [H_\tau] = 0 \text{ on } S \tag{1.9}$$

Here the last two equations apply if surface currents are absent, i.e. for finite conductivity of both mediums. A jump at S is defined here as $[a] = a - a_w$, where the index w refers to the body, and the indices n, τ refer to vector components normal to S and tangential to Σ, respectively. We will take the density of the surface distribution of fluid sources to be given.

Asymptotic conditions defining the behavior of p^*, u, h at infinite distance from the body must also be given. Analogous conditions for $\mathbf{V}_\perp$ and $\mathbf{H}_\perp$ play an auxiliary role; as will be shown, they cannot always be arbitrarily specified.

It is necessary to dwell in more detail on the continuity conditions for the electric field in (1.9), which, with the help of (1.2) and the other equations (1.9), may be put in the form

$$\nu_m\left(\frac{\partial H_n}{\partial \tau} - \frac{\partial H_\tau}{\partial n}\right) + v_n H_\tau = \frac{\mu_w}{\mu}\left[\nu_{mw}\left(\frac{\partial H_{nw}}{\partial \tau} - \frac{\partial H_{\tau w}}{\partial n}\right) + v_{nw} H_\tau\right]_{\text{on } S} \tag{1.10}$$

$$\nu_m \frac{\partial h}{\partial n} - v_n h = \frac{\mu_w}{\mu}\left(\nu_{mw}\frac{\partial h_w}{\partial n} - v_{nw}\, h\right) \tag{1.11}$$

The terms containing the transpiration velocity vanish for $q = 0$, $\mu_w = \mu$, and also for $\nu_{mw} \to 0$ (flow past a dielectric). For infinite conductivity of the body ($\nu_{mw} = 0$), it is, generally speaking, not possible to obtain in this way such simple boundary conditions as in problems without porosity [9].

2. Let us return now to the analysis of Equations (1.3) to (1.8). Cases are possible for which their solution reduces to a successive solution of very simple and even linear equations. This may occur, in particular, for $\mathbf{H}_\perp = \alpha\mathbf{V}_\perp$, when the transverse flow aligns itself with the lines of force of the transverse magnetic field. In fact, with this assumption, Equations (1.3) to (1.8) take the form

$$(\rho - \varkappa\alpha^2)(\mathbf{V}_\perp\nabla)\mathbf{V}_\perp - \varkappa\alpha\mathbf{V}_\perp(\mathbf{V}_\perp\nabla\alpha) - \eta\Delta\mathbf{V}_\perp = -\nabla' p^* \qquad (2.1)$$

$$\mathbf{V}_\perp(\mathbf{V}_\perp\nabla\alpha) = \nu_m[2(\nabla\alpha\nabla)\mathbf{V}_\perp + \alpha\Delta\mathbf{V}_\perp + \mathbf{V}_\perp\Delta\alpha] \qquad (2.2)$$

$$\begin{gathered}\mathbf{V}_\perp(\rho\nabla v - \varkappa\alpha\nabla h) = -\frac{\partial p^*}{\partial z} + \varkappa h\frac{\partial h}{\partial z} + \eta\Delta v \\ \mathbf{V}_\perp(\nabla h - \alpha\nabla v) = -v\frac{\partial h}{\partial z} + \nu_m\Delta h\end{gathered} \qquad (2.3)$$

$$\operatorname{div}\mathbf{V}_\perp = 0, \qquad \mathbf{V}_\perp\nabla\alpha = -0 \qquad (2.4)$$

Introducing the expressions for h from (1.8) into (2.3), and separating the z-terms, we obtain

$$\mathbf{V}_\perp(\rho\nabla v - \varkappa\alpha\nabla h_0) = \varkappa\theta h_0 + \eta\Delta v, \quad \mathbf{V}_\perp(\nabla h - \alpha\nabla v) = -v\theta + \nu_m\Delta h \quad (2.5)$$

$$\frac{\partial^2 p^*}{\partial z^2} = C = \varkappa(\theta^2 + \alpha\mathbf{V}_\perp\nabla\theta), \quad \mathbf{V}_\perp\nabla\theta = \nu_m\Delta\theta \qquad (2.6)$$

Thus, if $\mathbf{V}_\perp$ α, θ are found from Equations (2.2), (2.4), (2.6), so that a solution of Equation (2.1) for p^* exists (i.e. rot of its left-hand side becomes zero), then v and h_0 are subsequently determined from the linear equations (2.5). The system of equations with respect to $\mathbf{V}_\perp$ α, θ is, generally speaking, over-determined, and its solution may exist only in isolated cases, for which the form of the solution will at the same time indicate the possible form of the boundary conditions. It is not possible to reveal the totality of solutions here by elementary means, but, nevertheless, some important classes of solutions can be effectively studied - for example, for flows with pseudo-plane (in the sense of Barker) magnetic field, when $\theta = 0$, $\mathbf{H} = \mathbf{H}(x, y)$ and, in view of (2.4), $\alpha = \alpha(\psi)$, where ψ is the stream function of the transverse motion. Another series of solutions is obtained if it is assumed that $\mathbf{V}_\perp$ or $\mathbf{H}_\perp$ is a constant vector and the surface S is a plane, etc.

3. Let us investigate Equations (2.1) to (2.4) in more detail for the simplest case, when α = const. We note first that Equations (2.2) and (2.4) are more simply equivalent to

$$\operatorname{div} \mathbf{V}_\perp = 0, \qquad \operatorname{rot} \mathbf{V}_\perp = \Omega \mathbf{e}_z \tag{3.1}$$

where Ω is an arbitrary constant. The proof is obvious if the identity $\operatorname{rot}\operatorname{rot} \mathbf{V}_\perp = -\Delta \mathbf{V}_\perp$ is introduced into (2.2) and account is taken of the fact that $\mathbf{V}_\perp$ does not depend on z. Equation (1.2) may be expressed as follows:

$$\mathbf{j} = \frac{c}{4\pi}(\operatorname{rot} \mathbf{H}_\perp + \operatorname{rot} h\mathbf{e}_z) = \frac{c\alpha}{4\pi}\Omega \mathbf{e}_z + \left(\mathbf{e}_x \frac{\partial h}{\partial y} - \mathbf{e}_y \frac{\partial h}{\partial x}\right) \tag{3.2}$$

It is obvious that the vorticity Ω is proportional to the current density in the direction of the basic flow. It follows that in the absence of such a flow at infinite distance from the body the transverse flow must be potential.

Assume that the transpiration velocity v_n is given on S as a function of the arc length on Σ. If the stream function is introduced into (3.1) in the usual way, then with (1.9) we obtain the interior boundary value problem

$$\triangle\psi = -\Omega; \qquad \frac{\partial\psi}{\partial\tau} = f(s), \quad \frac{\partial\psi}{\partial n} = 0 \quad \text{on } \Sigma \tag{3.3}$$

which reduces in principle to the determination of the singularities of ψ at infinity.

An analogous interior problem appears in looking for the transverse magnetic field $\mathbf{H}_{\perp w}$ inside the body, if $\partial h_w/\partial z = 0$, the body is non-porous or $\mathbf{H}_{\perp w} = \alpha \mathbf{V}_{\perp w}$ inside it, and its conductivity is finite. In fact, in that case, in accordance with Section 1, $\Delta \mathbf{H}_{\perp w} = 0$, $\operatorname{div} \mathbf{H}_{\perp w} = 0$. Introducing the vector potential of the field, these equations give $\Delta A_w = -(4\pi/c)j_w$, where $j_w = \text{const}$, where the conditions on S (or, what is the same thing, on Σ) for A_w are obtained in the same way as in (3.3):

$$\begin{aligned} \frac{\partial A_w}{\partial\tau} &= H_{nw} = \frac{\mu}{\mu_w} H_n = \frac{\mu\alpha}{\mu_w} v_n = \frac{\mu\alpha}{\mu_w} f(s) \\ \frac{\partial A_w}{\partial n} &= -H_{\tau w} = -H_\tau = -\alpha V_\tau = 0 \end{aligned} \tag{3.4}$$

The determination of A_w reduces to the determination of the singularities or the magnetic field sources inside the body. If the body is porous and $\mathbf{H}_{\perp w} = \alpha \mathbf{V}_{\perp w}$, then a similar problem may also be formulated for the stream function ψ_w. It is evident that the inverse problem, to find ψ, A_w or ψ_w satisfying Poisson's equation and conditions of the type (3.3) for given singularities, may not have a solution, even if $f(s)$ is taken to be unknown.

Not dwelling further on these questions, we shall show that, in the majority of practically interesting cases, problems of type (3.3) for interior and exterior regions have a solution; furthermore, this solution satisfies (for $\Omega = 0$) such auxiliary conditions as boundedness of $\mathbf{V}_\perp$ and $\mathbf{H}_\perp$ at infinity. The inconvenience of the possible appearance of infinite values of $\mathbf{H}_{\perp w}$ inside the body is somewhat compensated by the fact that the solution as a whole gives a magnetic field normal to the surface, in the flow region which, in some sense, is the generalization of a homogeneous field.

We note also that, in view of Equations (1.10) and (3.2), the constant Ω and j_w are connected by the relation $\Omega = (4\pi\sigma_w / c a \sigma)$, which shows that, in the absence of a longitudinal current in the body, the transverse flow is potential. It is evident that the quantities Ω and j_w are not obtained from the solution of the problem, and one of them has to be given at the start.

Returning to Equations (2.1), (2.3) it is easy to see that for $\partial h/\partial z = 0$, $a = \text{const}$, and a not too rapid increase in the transverse velocity, all the terms in (2.3) vanish at infinity, from which $\partial p^*_\infty / \partial z = 0$. Integration of Equation (2.1), taking into account (3.1), now makes it possible to find the magnitude of the total pressure in the form

$$p^* = p^*_\infty + (\varkappa\alpha^2 - \rho)(\Omega\psi + \mathbf{V}_\perp{}^2/2), \qquad p^*_\infty = \text{const} \tag{3.5}$$

Introduce the dimensionless variables and parameters

$$x' = \frac{x}{L}, \quad y' = \frac{y}{L}, \quad v' = \frac{v}{v_0}, \quad \mathbf{V}_\perp{}' = \frac{\mathbf{V}_\perp}{v_0}, \quad h' = \frac{h}{\alpha v_0}, \quad \psi' = \frac{\psi}{Lv_0}$$

$$S = \frac{\varkappa\alpha^2}{\rho}, \quad R = \frac{v_0 L}{\nu}, \quad R_m = \frac{v_0 L}{\nu_m}, \quad N = \frac{\nu_m}{\nu} \qquad (v_0 > 0)$$

where L, v_0 are characteristic values of the body dimensions and the velocity. Then, omitting the primes on the dimensionless quantities, in place of (2.3) we obtain

$$R\mathbf{V}_\perp \nabla(v - Sh) = \triangle v, \qquad R_m \mathbf{V}_\perp \nabla(h - v) = \triangle h$$

Putting

$$u_i = v + \chi_i h, \qquad R_i = R_m (N - \chi_i) \tag{3.7}$$

$$\chi_{1,2} = \tfrac{1}{2}\left[N - 1 \pm \sqrt{(N-1)^2 + 4NS}\right]$$

it becomes possible to separate variables and to obtain for them the identical equations

$$R_i \frac{D(u_i, \psi)}{D(x, y)} = \triangle u_i \qquad (i = 1,2) \tag{3.8}$$

Such a transformation was used earlier in the solution of magnetohydrodynamic flow problems in the Oseen approximation [10]. It is useful if, from the initial conditions on v and h, it is possible to construct all the necessary conditions on u_i. A similar transformation of Equations (2.3) exists also in the more general case, where $a = a(\psi)$, $\partial p^*/\partial z \neq 0$.

Equations (3.8) preserve their form in going over to an arbitrary isothermal coordinate system $\zeta_1(x, y)$, $\zeta_2(x, y)$, characterized by the equations $\Delta\zeta_1 = \Delta\zeta_2 = \nabla\zeta_1 \nabla\zeta_2 = 0$, i.e.

$$R_i \frac{D(u_i, \psi)}{D(\zeta_1, \zeta_2)} = \frac{\partial^2 u_i}{\partial \zeta_1^2} + \frac{\partial^2 u_i}{\partial \zeta_2^2} \qquad (i = 1,2) \tag{3.9}$$

If the transverse flow is irrotational ($\Omega = 0$) and its complex potential is $\Psi = \phi + i\psi$, then we may put $\zeta_1 = \phi$, $\zeta_2 = \psi$ and

$$R_i \frac{\partial u_i}{\partial \varphi} = \frac{\partial^2 u_i}{\partial \varphi^2} + \frac{\partial^2 u_i}{\partial \psi^2} \qquad (i = 1, 2) \tag{3.10}$$

A similar simplification can be obtained also in the case of rotational flow, when the streamlines coincide with streamlines of a potential flow, i.e., for $\psi = \psi(\zeta_2)$. From Equation (3.3) it follows that in this case $\psi''(\zeta_2)(\nabla\zeta_2)^2 = -\Omega$, and, in accordance with the results of [11], for $\Delta\zeta_2 = 0$ and $(\nabla\zeta_2)^2 = f(\zeta_2)$, the lines $\zeta_2 = \text{const}$ map a family of either parallel straight lines or concentric circles. Consequently, for $\Omega = 0$ it is possible to investigate problems of longitudinal flow past a plate and a double wedge, for which (3.9) takes the form

$$R_i \frac{d\psi}{d\zeta_2} \frac{\partial u_i}{\partial \zeta_1} = \frac{\partial^2 u_i}{\partial \zeta_1^2} + \frac{\partial^2 u_i}{\partial \zeta_2^2} \qquad (i = 1, 2)$$

For all other surfaces, for $\Omega \neq 0$, a coordinate transformation of the type written above allows the construction of an integral equation equivalent to (3.9), if the Green's function is known for the given boundary-value problem in potential flow (cf., for example, [12]). Due to the fact that the condition of boundedness of $\mathbf{V}_\perp$ and $\mathbf{H}_\perp$ at infinity is not met, in general, for a flow with constant vorticity, these problems are of comparatively little interest.

In the case of potential transverse flow, Equations (3.10) have the particular solution

$$u_i(\varphi) = C_i + D_i \exp R_i \varphi \qquad (i = 1, 2) \tag{3.11}$$

where C_i, D_i are constants. It is suitable for problems in which the boundary values and the asymptotic behavior of u_i do not depend on ψ. Having constructed a simple solution of the flow past a flat plate ($\phi = \pm y$, $\psi = \pm x$, $v_y = \pm 1$), it is not difficult, on the basis of

Equations (3.11), to study the flow past other cylindrical bodies with the help of coordinate transformations.

4. Let us conclude with a consideration of some peculiarities of the flow past dielectric bodies, when the transverse flow is known to be potential. From the equations $\mathbf{j}_w = (c/4\pi)\ \mathrm{rot}\ \mathbf{H}_w = 0$, $\nu_{mw} \to 0$ it is easy to conclude that inside the body $h_w = \mathrm{const}$ and condition (1.11) is satisfied automatically. Consequently, $v = v_w$, $h = h_w$ on the surface, or

$$u_i = v_w + \chi_i h_w = u_{iw} \text{ on } \Sigma \quad (i = 1, 2) \tag{4.1}$$

Assuming that $v \to v_\infty = \mathrm{const}$, $h \to h_\infty = \mathrm{const}$ at infinite distance from the body i.e.

$$u_i \to v_\infty + \chi_i h_\infty = u_{i\infty} \quad (i = 1, 2) \tag{4.2}$$

we may make use of the solution (3.11). We note now that $u_i(\phi)$ and ϕ take on constant values on Σ; furthermore, ϕ has no singularities at finite distances from the body. Therefore, in view of the transformations of Section 3, it may be concluded that the streamlines $\psi = \mathrm{const}$ go to infinity, and along them ϕ grows without bound in absolute value: $\lim \phi = \pm\infty$ for $\sqrt{(x^2 + y^2)} \to \infty$. The upper and lower signs here correspond to blowing and sucking at the surface of the body. Thus it is possible to investigate the asymptotic behavior of the solution (3.11) without going over to the primary coordinate system.

It is immediately discovered that conditions (4.2) can be fulfilled only for $R_{1,2}\,\phi < 0$. As is clear from (3.7), a flow satisfying the four independent conditions (4.1), (4.2) exists only for suction with super-Alfven velocity ($\zeta < 1$, $\phi < 0$). If the velocity of the transverse flow in sub-Alfven ($\zeta > 1$), then, for suction and for blowing, one of the quantities $R_i\phi$ will be positive, and of the conditions (4.1), (4.2) only three can be independent. Finally, for blowing with super-Alfven velocity, $R_{1,2} > 0$, and then only quasi-solid body motion is possible, with $v \equiv v_w$, $h \equiv h_w$. It should be emphasized that only the solutions with $\zeta < 1$ are continuous generalizations of the results of ordinary hydrodynamics, in which $\zeta = 0$.

In conclusion, we shall calculate the drag of the dielectric body per unit length:

$$F = \oint_\Sigma \left(\eta \frac{\partial v}{\partial n} + \varkappa H_n h \right) d\Sigma \tag{4.3}$$

Going to dimensionless quantities and coordinates ϕ, ψ, we obtain

$$C_f = \frac{F}{L\rho r_0^2} = \oint_\Sigma \left(\frac{1}{R} \frac{\partial v}{\partial \varphi} + Sh \right) d\psi \tag{4.4}$$

For solutions of type (3.11) the function in the integral does not depend on ψ and, consequently

$$C_f = \left(\frac{1}{R}\frac{\partial r}{\partial \varphi} + Sh\right)_\Sigma Q \qquad \left(Q = \oint_\Sigma d\psi\right) \tag{4.5}$$

where Q is the total dimensionless strength of the fluid sources on the surface of the body and in its interior, per unit length.

It should be remembered that the discussion of Section 4 makes sense only when along the surface contour the transpiration velocity v_n does not change its direction with respect to the normal and is not identically equal to zero.

Limiting cases of longitudinal flow — without magnetic field with potential transverse flow, and without transverse flow (non-porous surface) with transverse potential field — may be obtained, respectively, for $a \to 0$ and $V_\perp \to 0$, $V_\perp \alpha \to H_\perp$. However, in this case, many of the formulas (especially Sections 3, 4) will undergo fundamental changes, and therefore these cases are best investigated directly on the basis of Equations (1.3) to (1.8).

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